



Electrohydrodynamic stability of poorly conducting parallel fluid flow in the presence of transverse electric field

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Abstract

In this paper we study electrohydrodynamic instability (EHDI) in a poorly conducting parallel inviscid fluid in the presence of an electric field and space variation of electrical conductivity. It is shown that EHDI causes inhomogeneity in the material science processing. This inhomogeneity can be controlled by understanding the nature of EHDI in the presence of an electric field and a shear due to horizontal basic velocity. The condition for EHDI is determined in terms of the electric number rather than the point of inflexion of the basic velocity profile using both moment and energy methods combined with Galerkin expansion technique. From this analysis, it is shown that a proper choice of electric number controls inhomogeneity by controlling instability of a parallel poorly conducting inviscid fluid. For unstable motion it is shown that the growth rate, C_i , is confined in a semi circle region

$$C_r^2 + C_i^2 - 2 \left(u_b - \frac{\Omega}{\phi} \right) C_r + u_b^2 - 2u_b \frac{\Omega}{\phi} = 0,$$

which has the center $(u_b - \Omega/\phi, 0)$ and radius $|\Omega/\phi|$ where C_r is the phase velocity, u_b the basic horizontal velocity, $\phi = D^2 u_b$, $\Omega = W_1 x_0^2 \alpha^2$ the electric number and $D = d/dy$. From this an upper bound for the amplification factor is shown to be as

$$C_i^2 \leq \max \left| \frac{\Omega}{\phi} \right|^2,$$

under the condition that ϕ has the same sign between 0 and 1.

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1. Introduction and motivation

It is well known (see [1]) that in Fluid Mechanics the physics of a steady flow cannot be fully understood without examining its stability. The same is also true for electrohydrodynamics (EHDs) which is the study of an interaction of a poorly

conducting fluid in the presence of an electric field involving the effect of fluid motion on the fields and the influence of the fields on the motion. In modern technological world these two effects are important in miniaturizing the devices by increasing the efficiency and decreasing the weight. Therefore, there is a great demand for new kinds of pumps and generators using electric traveling waves as the conveyor of liquids. The available literature [2,3] in these considerations has used a dielectric liquid as the media to propel. These dielectric liquids which have the properties of piezoelectric are ideal for ultrasonic applications due to high frequency response but they may not be

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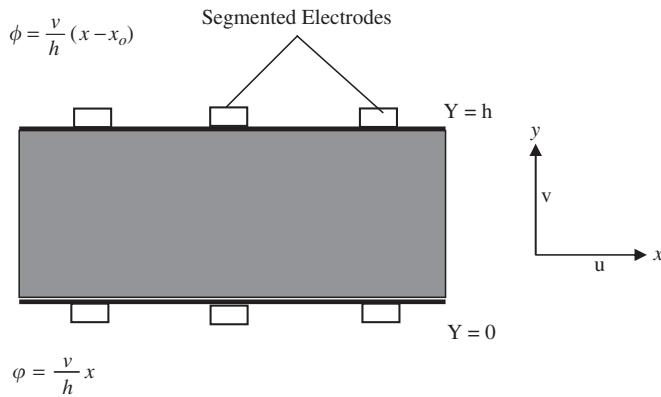


Fig. 1. Physical configuration.

of much use for bio-engineering and for some industrial applications. (see [4]). Further, dielectric liquids exhibit anisotropy and inhomogeneity leading to complications in the development of suitable theory. One of the objectives of this paper is to overcome these complications using smart materials made up of the poorly conducting liquids like mixture of nickel–titanium shape memory alloys and so on. In the process of synthesizing these smart materials the electrical conductivity of these poorly conducting liquids varies with the mixture of these alloys increasing with height. The variation of electric conductivity σ with height releases charges which in turn produce an electric field called induced electric field. In addition, there may be an applied electric field due to embedded electrodes at the surface (see Fig. 1). This total electric field not only produces a current which acts as sensing like piezoelectric current in dielectrics but also produces an electrostatic force which acts as actuator. These two properties are essential for a material to be a smart material.

It is known that (see [1]) the EHD steady flows become more unstable than in the ordinary steady flows, because of the existence of the fine-scale turbulence when a fluid volume is stressed by electric fields. These fine scale turbulence need high Reynolds number having low viscosity effect. In addition, some of the poorly conducting alloys will have not only infinitesimally small electrical conductivity but also have low viscosity. In this regard we assume the fluid to be non-viscous. These fine scale turbulence causes inhomogeneity in the process of synthesizing smart materials. The effective functioning of smart materials depends on a mechanism to suppress inhomogeneity by controlling the fine scale turbulence. This can be achieved using stability analysis, which is also one of the objectives of this paper. In ordinary fluids, the hydrodynamic stability of homogeneous or heterogeneous inviscid or viscous fluid has been extensively investigated in the literature using Orr–Sommerfeld equation in viscous fluid and Rayleigh equation in inviscid fluid (see [5–8]), because of its importance in understanding the characteristics of fluid flows particularly in understanding the transition from laminar to turbulent flow. The hydrodynamic stability of heterogeneous inviscid fluid has also been investigated by Taylor [9], Goldstein [10] and Synge [11] because of its importance in atmospheric and

oceanographic sciences. Taylor [9] and Goldstein [10] have found the condition for stability using the energy method while Synge [11] found, treating the problem as boundary value problem, $J_H > \frac{1}{4}$ to be a sufficient condition for stability where J_H is the Richardson number. The hydrodynamic stability of heterogeneous inviscid fluid has also been investigated by Miles [12] and Howard [13]. Later these hydrodynamic stability problems have been extended to hydromagnetic stability (see [14,15]) with the objective of using magnetic field to suppress the onset of instability. Later Rudraiah (see [16–18]) has extended the work of Synge to electrically conducting fluid in the presence of a transverse magnetic field and showed that the condition for stability of heterogeneous inviscid conducting fluid to be $J > \frac{1}{4}$ where J is the sum of hydrodynamic and hydromagnetic Richardson numbers. We note that in hydrodynamic stability of heterogeneous inviscid fluid the ratio of layer stratification to velocity shear is a dominating force, whereas in hydromagnetic stability the Lorentz force is the dominant force in addition to ratio of layer stratification to velocity shear. The hydrodynamic and magnetohydrodynamic surface instabilities of the type Rayleigh–Taylor (RTI) occurring when heavy fluid is supported by lighter fluid, Kelvin–Helmholtz instability (KHI) at the interface between two moving fluids caused by shear and Richtmyer–Meshkov instability (RMI) at the interface between two fluids caused by shock wave have been investigated in the literature (see [19]) because of their importance in material sciences. In addition to these instabilities there is another type of instability called electrohydrodynamic instability (EHDI), the name given to those instabilities occurred in a poorly conducting fluid in the presence of an electric field when the spatial gradients of electrical properties such as electrical conductivity σ , electric permittivity (that is dielectric constant) ϵ exist. The classical examples of EHDI are those that occur at the boundary between immiscible poorly conducting liquids, piezoelectric material and so on. A review of the role of interfacial stresses on EHDs has been given by Melcher and Taylor [20] and later the same aspect has been investigated by several other authors (see [1,21–24]). Their work has been concerned with convective EHD instabilities, but the EHD stability in a poorly conducting parallel flow in the presence of an electric field has not been given much attention in spite of its importance in the problems mentioned above with the variation of electrical conductivity due to layer stratification, concentration of mixture of alloys, difference in temperature and so on. Recently, Baygents and Baldessare [25] have investigated EHD stability in a thin fluid layer in the presence of a gradient of electrical conductivity assuming the base flow to be quiescent. Many practical problems mentioned above, particularly the nature of smart materials in atmospheric and geophysical process, involve the basic flow in a poorly conducting inviscid fluid in the presence of an electric field and the gradient of electrical conductivity. As in the classical problem its stability is known as EHDI. These types of instabilities can be treated by means of various methods. The simplest approach is to use the Normal mode technique which is restricted to small amplitude disturbances and yields a sufficient condition for stability, which is obtained using moment and energy methods. These methods

are useful in studying the stability with respect to small as well as large disturbances (i.e. universal stability). They are the global methods where the details of motion and flow geometry are ignored. Instead, attention is focused on the behavior of global quantities generally chosen as some positive definite functionals of the disturbances. In its simplest form proposed by Reynolds [26] and Orr [27] the functional was momentum in the moment method and kinetic energy of the disturbances in the energy method. These methods have been extended to magnetohydrodynamic stability and stability of flow through porous media by Rudraiah [28].

This EHDI also plays a significant role in several atmospheric and geophysical processes. For example, transition between ionosphere and atmosphere of the earth is a region having poorly conducting fluid in which the electrical force dominates in driving the fluid (see [29]). Further, the electrical forces in thunderstorm also appear to be as important as thermal forces at some stages of their development. There is a considerable empirical evidence that Electrohydrodynamic flows, for example in pumps and generators or image processing devices involving basic shear flow in the presence of an electric field which are considered as even likely to be more unstable than ordinary steady flow (see [1]) because of the reason stated earlier. However, the Electrohydrodynamic stability (EHDs) in a viscous or non-viscous poorly conducting basic shear flow has not been given much attention in the literature inspite of its importance in astrophysics, geophysics involving smart materials free from inhomogeneity (see [30]). The study of it is the main objective of this paper considering poorly conducting inviscid fluid with basic shear. In a poorly conducting fluid the electrical conductivity σ is a function of layer stratification and increases with an increase in vertical distance that is $\sigma = \sigma(y)$. If σ is a constant, we show that $\rho_e = 0$ and hence no electric field, with the result no current that is no sensing and no electric force that is no actuation. In other words if $\sigma = \text{Constant}$, the material will not have the properties of smartness. Therefore, in this paper to ensure the material work as smart material, we consider the variation σ with the vertical distance y .

To achieve this objective, the plan of this paper is as follows. The required basic equations and the boundary conditions are given in Section 2. The electric potential is obtained in Section 3 by solving the Maxwell's equations for a poorly conducting fluid with segmented electrodes at the boundaries. The condition for stability or instability of a parallel poorly conducting fluid in the presence of an electric field is discussed in Section 4 using both the moment and energy methods together with Gelarkin expansion. An upperbound for the amplification factor for instability is obtained and it is shown that the growth rate C_i is confined in a semi circle region. Some important conclusions are drawn in the final Section 5.

2. Formulation of the problem

We consider an infinite horizontal poorly conducting fluid layer bounded on both sides by electro conducting impermeable rigid plates embedded with segmented electrodes located at $y=0$ and at $y=h$ (see Fig. 1) having different electric potentials

$\phi = (v/h)x$ and $\phi = (v/h)(x - x_0)$ maintained at these boundaries as shown in Fig. 1. The basic equations for incompressible poorly conducting non viscous homogeneous fluids are

$$\text{conservation of mass: } \nabla \cdot \vec{q} = 0, \quad (1)$$

conservation of momentum:

$$\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla) \vec{q} = -\frac{1}{\rho_0} \nabla p + \frac{\rho_e}{\rho_0} \vec{E}, \quad (2)$$

conservation of electric charges:

$$\frac{\partial \rho_e}{\partial t} + (\vec{q} \cdot \nabla) \rho_e + \nabla \cdot (\sigma \vec{E}) = 0, \quad (3)$$

$$\text{Ohm's law: } \vec{J} = \sigma \vec{E}. \quad (4)$$

In this paper, we use the EHD approximations namely the electrical conductivity, σ , of the liquid is negligibly small (because we consider a poorly electrically conducting liquid) so that the induced magnetic field is negligible. Further, there is no applied magnetic field. This approximation makes the electric field \vec{E} , to be conservative. The Maxwells equations under these approximations are

$$\text{Gauss law: } \nabla \cdot \vec{E} = \frac{\rho_e}{\epsilon_0}, \quad (5)$$

$$\text{Faradays law: } \nabla \times \vec{E} = 0, \quad \vec{E} = -\nabla \phi. \quad (6)$$

The applied electric field, \vec{E}_a , due to embedded electrodes is in the vertical y direction. We assume two dimensional flow given by $\vec{q} = (u, v) \vec{E} = (E_x, E_y)$ and $\nabla = (\partial/\partial x) \hat{i} + (\partial/\partial y) \hat{j}$ where (u, v) and (E_x, E_y) are, respectively, the components of velocity and electric field in x and y directions.

Under these approximations the basic equations (1)–(6) for a poorly electrically conducting, non-viscous, incompressible two dimensional homogeneous fluid, after making them dimensionless using the scales h for length, U for velocity, $\rho_0 U^2$ for pressure, $\epsilon_0 V/h^2$ for density of charge ρ_e , V for potential, V/h for electric field, σ_0 for conductivity, ρ_0 the density of fluid, ϵ_0 the dielectric constant, take the form

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (7)$$

$$\frac{Du}{Dt} = -\frac{\partial p}{\partial x} + W_1 \rho_e E_x, \quad (8)$$

$$\frac{Dv}{Dt} = -\frac{\partial p}{\partial y} + W_1 (\rho_e E_y), \quad (9)$$

$$\frac{D\rho_e}{Dt} + \frac{\partial \sigma}{\partial x} E_x + \frac{\partial \sigma}{\partial y} E_y = 0, \quad (10)$$

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} = \rho_e, \quad (11)$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = 0, \quad (12)$$

where p is the pressure, \vec{J} the current density, ρ_e the density of electric charges, $W_1 = \epsilon_0 v^2 / \rho_0 U^2 h^2$ the electric number which

represents physically the ratio of electric energy to kinetic energy, $R = \varepsilon_0 U / \sigma_0 h$ is the dimensionless relaxation time,

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} \quad \text{and} \quad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}.$$

The boundary conditions, in dimensionless form, are

$$u, v = 0 \quad \text{at} \quad y = 0 \quad \text{and} \quad 1, \quad (13)$$

$$\phi = x \quad \text{at} \quad y = 0,$$

$$\phi = x - x_0 \quad \text{at} \quad y = 1. \quad (14)$$

The variation of σ with layer stratification releases the distribution of density of charges which in turn produce an induced electric field, E_i , called stratified electric field. The total electric field $\vec{E} (= \vec{E}_a + \vec{E}_i)$ produces a current which acts as sensing and also produces an electric force which acts as actuation. These two are the important properties of a smart material. If the electrical conductivity is constant, the charges will not be released that is $\rho_e = 0$ as shown in the following Result.

Result 1. In a poorly conducting fluid if its electrical conductivity is a constant then the material does not function as a smart material.

Proof. The Gauss Law, Eq. (5) using Eq. (6), takes the form

$$\varepsilon_0 \nabla^2 \phi = -\rho_e, \quad (15)$$

where the density of distribution of charges ρ_e is given by the conservation of charges,

$$\frac{\partial \rho_e}{\partial t} + \nabla \cdot \vec{J} = 0, \quad (16)$$

where

$$\vec{J} = \sigma \vec{E} + \rho_e \vec{q}. \quad (17)$$

On the RHS of Eq. (17) the first term is the conduction current and the second term is the convective current. Since the poorly conducting fluid is incompressible, we have

$$\nabla \cdot \vec{J} = \nabla \cdot (\sigma \vec{E}) + (\vec{q} \cdot \nabla) \rho_e. \quad (18)$$

Then Eq. (16), using Eq. (18), becomes

$$\frac{\partial \rho_e}{\partial t} + (\vec{q} \cdot \nabla) \rho_e + \sigma \nabla \cdot \vec{E} + \vec{E} \cdot \nabla \sigma = 0. \quad (19)$$

This, using Eq. (15), Eq. (6) and $\vec{q} = u \hat{i}$, becomes

$$\varepsilon_0 \left(\frac{\partial}{\partial t} \right) \nabla^2 \phi + \varepsilon_0 u \frac{\partial}{\partial x} (\nabla^2 \phi) + \sigma \nabla^2 \phi + \nabla \phi \cdot \nabla \sigma = 0. \quad (20)$$

If σ is a constant then Eq. (20) becomes

$$\left(\varepsilon_0 \frac{\partial}{\partial t} + \varepsilon_0 u \frac{\partial}{\partial x} + \sigma \right) \nabla^2 \phi = 0. \quad (21)$$

We look for the solution of Eq. (21) in the form

$$\phi = \phi_0(x, y, z) e^{i(\omega t + lx)}, \quad (22)$$

where ω is the frequency.

Eq. (21), using (22), becomes,

$$(i\omega \varepsilon_0 + i l \varepsilon_0 u + \sigma) D^2 \phi_0 = 0, \quad (23)$$

where $D^2 = (\partial^2 / \partial y^2) - l^2$.

Since $(i\omega + i l u + (\sigma / \varepsilon_0)) \neq 0$ we have $\varepsilon_0 \nabla^2 \phi_0 = 0$.

Therefore, using Eq. (15) we get $\rho_e = 0$.

This implies that there is no induced electric field with the result there is no current to act as sensing and there is no electric force $\rho_e \vec{E}$ to act as actuation. This shows that if σ is a constant the poorly conducting material will not function as a smart material. Hence the result. \square

Our aim in this paper is to develop a mathematical model to synthesize a smart material using a poorly conducting fluid free from the inhomogeneity by reducing the inhomogeneity in the end product of smart material by controlling instabilities caused by the electric field. For this purpose we have to consider σ not to be a constant, and varies with position.

3. Basic state

We consider the following basic state:

$$u = u_b(y), \quad \sigma = \sigma_b(y), \quad \rho_e = \rho_{eb}(y), \quad \vec{E} = E_b \hat{j}, \quad (24)$$

where the suffix b denotes the basic state. Substituting Eq. (24) into Eqs. (1)–(6) and then simplifying we get

$$\frac{\partial^2 \phi_b}{\partial y^2} + \alpha \frac{\partial \phi_b}{\partial y} = 0, \quad (25)$$

where $\alpha = \alpha_h h$ is the volumetric coefficient of conductivity and $\rho_{eb} = -\partial^2 \phi_b / \partial y^2$.

Solution of Eq. (25), using the boundary conditions Eq. (14), is

$$\phi_b = x - \frac{x_0(1 - e^{-\alpha y})}{(1 - e^{-\alpha})}. \quad (26)$$

As stated earlier the EHD flows in practical problems like propulsion in pumps, generators and so on involving both electric fields and fluids are considered to be more unstable than ordinary fluids in the absence of an electric field. To synthesize smart materials free from inhomogeneity the system has to be stable. To know this we have to investigate stability analysis of a perturbation caused by electric field on the base flow of a poorly conducting fluid. Such stabilities are known as EHDS. At present these EHDS are concerned with surface instabilities like Rayleigh–Taylor, Kelvin–Helmholtz, Richtmyer–Meschov types in a poorly conducting fluid (see [16]). But much attention has not been given to EHDS of parallel flows, either in non-viscous fluid governed by modified Rayleigh equation or in a viscous fluid governed by modified Orr–Sommerfeld equation. Therefore, the objective of this paper is to study the linear stability of a poorly conducting inviscid parallel flow as explained in next section.

4. Stability equations

To study the linear stability of the basic state (24) we superimpose on it an infinitesimal symmetrical disturbances of the form

$$\begin{aligned} u &= u_b + u', \quad v = v', \quad p = p_b + p', \quad \vec{E} = \vec{E}_b \hat{j} + \vec{E}', \\ \rho_e &= \rho_{eb} + \rho_e', \end{aligned} \quad (27)$$

where the primes denote the perturbed quantities which are assumed to be infinitesimally small compared to the basic state. Substituting Eq. (27) into Eqs. (1)–(4) and linearizing them by neglecting the product of higher order terms in perturbed quantities and for simplicity neglecting the primes we get

$$\frac{\partial u}{\partial t} + u_b \frac{\partial u}{\partial x} + v \frac{\partial u_b}{\partial x} = -\frac{\partial p}{\partial x} + W_1 \rho_{eb} E_x, \quad (28)$$

$$\frac{\partial u}{\partial t} + u_b \frac{\partial v}{\partial x} = -\frac{\partial p}{\partial y} + W_1 (\rho_{eb} E_y + \rho_e E_{by}). \quad (29)$$

We define the stream function ψ satisfying Eq. (7) by

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}. \quad (30)$$

Eliminating the pressure between Eqs. (28) and (29) and using Eq. (30) and using the normal mode solution of the form

$$f'(x, y, t) = f(y)e^{il(x-Ct)}, \quad (31)$$

where l is the wave number and $C = n/l$, the wave velocity, n is the frequency, we get the stability equation

$$-(D^2 - l^2)\psi + \frac{D^2 u_b \psi}{(u_b - C)} - \frac{W_1 x_0 (D^2 - l^2)\phi}{(u_b - C)} = 0. \quad (32)$$

Similarly, the equation of continuity of charges takes the form

$$[il(u_b - C)_3](D^2 - l^2)\phi = il\alpha^2 x_0 \psi. \quad (33)$$

Substituting $(D^2 - l^2)\phi$ from Eq. (33) into Eq. (32) and after simplification we get

$$D^2 \psi - l^2 \psi - \frac{D^2 u_b \psi}{(u_b - C)} + \frac{W_1 x_0^2 \alpha^2 \psi}{(u_b - C)^2} = 0. \quad (34)$$

This is a modified form of Rayleigh equation, modified in the sense of incorporating the contribution from the electric force, $\rho_e \vec{E}$, which is the last term on the left-hand side of Eq. (34).

In hydrodynamic inviscid homogeneous fluid, Rayleigh [31] had proved that “A necessary condition for instability is, the basic velocity profile should have an inflexion point” (see [4]). However, in EHDs the nature of instability discussed here is quite contrast to the Rayleigh’s inflexion point as proved in the following result.

Result 2. A necessary condition for EHDI in an inviscid, homogeneous, poorly conducting fluid, instability in the

absence of Rayleigh inflexion point ($D^2 u_b \neq 0$) and the basic velocity u_b does not balance the phase velocity C_r in the open interval $(0, 1)$, is

$$W_1 = \frac{D^2 u_b}{2x_0 \alpha^2 (u_b - C_r)[(u_b - C_r)^2 + C_i^2]}. \quad (35)$$

Proof. To prove this result, first we have to obtain a compatibility condition using moment and energy methods together with the single term Galerkin expansion technique of the form

$$\psi = A\psi_1, \quad \phi = B\phi_1, \quad (36)$$

where ψ_1 and ϕ_1 are trial functions. A and B are known constants. To apply momentum or energy method we first multiply Eq. (32) by ψ^m and Eq. (33) by ϕ^m and substituting Eq. (36) in Eq. (32) and (33), eliminating A and B and integrating the resulting equation from 0 to 1 with respect to y , we get

$$\int_0^1 \left[\frac{-(D^2 - l^2)\psi\psi^m(u_b - C)^2 + D^2 u_b \psi\psi^m(u_b - C) - W_1 x_0^2 \alpha^2 \psi\psi^m}{(u_b - C)^2} \right] \times dy = 0. \quad (37)$$

Here $m=0$ corresponds to moment method, $m=1$ corresponds to energy method and $C = C_r + iC_i$. The nature of solution (31) reveals that if $C_i > 0$, the system is unstable, and neutrally stable if $C_i = 0$.

Rationalizing Eq. (37) and considering the imaginary part we get for $m=1$,

$$C_i \int_0^1 \left[\frac{D^2 u_b \psi\psi^m[(u_b - C_r)^2 + C_i^2] - 2W_1 x_0^2 \alpha^2 \psi\psi^m(u_b - C_r)}{[(u_b - C_r)^2 + C_i^2]^2} \right] \times dy = 0, \quad (38)$$

which is the Modified Rayleigh’s stability equation.

From Eq. (38) it follows that, either C_i is zero if the integral is not zero or C_i is not zero if the integral is zero. Since W_1 and other quantities except $D^2 u_b$ and $(u_b - C_r)$ are always positive, we have

$$W_1 = \frac{D^2 u_b}{2x_0 \alpha^2 (u_b - C_r)[(u_b - C_r)^2 + C_i^2]}.$$

Hence the result. \square

Corollary. In EHD inviscid homogeneous flow the system is neutrally stable if

$$W_1 \neq \frac{D^2 u_b}{2x_0 \alpha^2 (u_b - C_r)[(u_b - C_r)^2 + C_i^2]} \quad (39)$$

and $D^2 u_b$ has the same sign in the interval $(0, 1)$ and $u_b \neq C_r$.

This corollary follows from Eq. (38) using the conditions $D^2 u_b \neq 0$ and $u_b \neq C_r$.

From this Result 2 and its Corollary it follows that the nature of stability in EHDs is controlled by electric number W_1 rather than the inflexion point in the basic velocity profile.

Result 3 (Semicircle Theorem in EHD). For unstable waves C must lie in the semicircle

$$C_r^2 + C_i^2 - 2 \left(u_b - \frac{\Omega}{\phi} \right) C_r + u_b^2 - 2u_b \frac{\Omega}{\phi} = 0. \quad (40)$$

Proof. If $C_i \neq 0$ that is if the motion is unstable and $D^2 u_b \neq 0$, $W_1 \neq 0$, $u_b \neq C_r$ in the range 0,1. Eq. (38) gives,

$$D^2 u_b [(u_b - C_r)^2 + C_i^2] - 2W_1 x_0^2 \alpha^2 (u_b - C_r) = 0.$$

Simplifying and rearranging, we get Eq. (40).

In the complex plane C , let L be any point in the range (0, 1) such that Eq. (40) is satisfied at that point. Eq. (40) represents the family of circles in the complex plane with

$$\text{center} = \left(u_b - \frac{\Omega}{\phi}, 0 \right), \quad (41)$$

$$\text{radii} = \left| \frac{\Omega}{\phi} \right| \quad (42)$$

and they cut the real axis at the points

$$(u_b, 0) \quad \text{and} \quad \left(u_b - \frac{2\Omega}{\phi}, 0 \right), \quad (43)$$

where

$$\Omega = W_1 x_0^2 \alpha^2, \quad \phi = D^2 u_b. \quad (44)$$

Hence the result. \square

Let the imaginary part of C be confined in the region Σ , which is being composed of singly infinite family of circles Eq. (40) and each of these circles corresponding to a point L of the fluid. Σ has two regions common with the C_r -axis, either separate or overlapping. Of these, one extends between the value of C_r equal to the minimum or maximum values of u_b and the other between the values of C_r equal to the minimum or the maximum value of $(u_b - (2\Omega/\phi))$.

Further, either from Eq. (40) because the radius of it is $|\Omega/\phi|$ or directly from Eq. (38) under the condition that ϕ has the same sign between 0 and 1, we have

$$C_i^2 \leq \text{Max} \left| \frac{\Omega}{\phi} \right|^2. \quad (45)$$

This inequality equation (45) gives an upperbound for the amplification factor and it follows that every positive C_i is always associated with a negative one of the same magnitude.

A stronger form of Result 2 can be obtained by following the works of *Fj\phi ortoft* (see [5]) as in Result 4.

Result 4. A necessary condition for EHD instability is that $D^2 u_b$ and $(u_b - C_r)$ are different from zero and are of the same sign and

$$W_1 > \frac{D^2 u_b [(u_b - C_r)^2 + C_i^2]}{x_0^2 \alpha^2 (u_b - C_r)}. \quad (46)$$

Proof. Following *Fj\phi ortoft*, from the real part of Eq. (34) with $m = 1$, we get

$$\int_0^1 \left\{ \frac{D^2 u_b (u_b - C_r)}{[(u_b - C_r)^2 + C_i^2]} - \frac{W_1 x_0^2 \alpha^2 [(u_b - C_r)^2 - C_i^2]}{[(u_b - C_r)^2 + C_i^2]^2} \right\} \times |\psi|^2 dy = - \int_0^1 (|D\psi|^2 + l^2 |\psi|^2) dy < 0. \quad (47)$$

We now add,

$$- \int_0^1 C_i^2 \frac{W_1 x_0^2 \alpha^2}{[(u_b - C_r)^2 + C_i^2]^2} |\psi|^2 dy = 0$$

to left-hand side of Eq. (47) and obtain

$$\int_0^1 \left\{ \frac{D^2 u_b (u_b - C_r)}{[(u_b - C_r)^2 + C_i^2]} - \frac{W_1 x_0^2 \alpha^2 (u_b - C_r)^2}{[(u_b - C_r)^2 + C_i^2]^2} \right\} \times |\psi|^2 dy = - \int_0^1 (|D\psi|^2 + l^2 |\psi|^2) dy < 0.$$

Simplifying we get Eq. (46) from which the result follows. Hence the result. \square

5. Conclusions

Eq. (35) can be written as

$$\frac{d^2 \psi}{dy^2} - \chi(y) \psi = 0, \quad (48)$$

where

$$\chi(y) = l^2 + \frac{D^2 u_b}{(u_b - c)} - \frac{W_1 x_0^2 \alpha^2}{(u_b - c)^2}. \quad (49)$$

From the comparison theorem, it is known that if, throughout the range 0–1,

$$\chi(y) > -\pi^2, \quad (50)$$

then there does not exist a solution of Eq. (46) satisfying the condition Eq. (13). By Eq. (50), considering $\chi(y)$ as a function of C in the range (0, 1), it follows that for

$$C = \pm\infty, \quad \chi(y) = l^2. \quad (51)$$

For $C = u_b$, χ goes to infinity which is a singularity of Eq. (48) and known as resonance. Also,

$$\frac{\partial \chi}{\partial C} = \frac{P}{(u_b - C)^2} - \frac{2q}{(u_b - C)^3}, \quad (52)$$

which vanishes for

$$C = u_b - \frac{2\Omega}{\phi}, \quad (53)$$

where Ω and ϕ are as defined in Eq. (44).

We can see that $\chi(y)$ has maximum at C given by Eq. (53) and its maximum value is

$$\chi = l^2 + \frac{\phi^2}{4\Omega}. \quad (54)$$

From Eq. (54) it is evident that if

$$\frac{\phi^2}{4\Omega} > -\pi^2, \quad (55)$$

then Eq. (48) is satisfied at the point in question no matter what real values l and c may have. Hence, throughout the liquid, if

$$0 < \frac{D^2 u_b}{W_1 x_0^2 \alpha^2} < 4\pi^2, \quad (56)$$

then there is instability.

In the case of steady uniform velocity, that is $u_b = \text{constant}$, $\phi = 0$ and $\Omega \neq 0$, the circle Eq. (40) degenerates into a straight line in the C -plane with the equation $u_b = C_r$ and there is no complex region. Hence the motion is stable for $u_b = \text{constant}$ and $\Omega \neq 0$. This clearly shows that the effect of electric field makes the flow stable. If $\phi \neq 0$ and $\Omega = 0$ then Eq. (38) shows that the motion is stable anywhere in the range 0–1. If $D^2 u_b = 0$ (i.e. inflexion point of the basic velocity profile) somewhere in the range 0–1 then the motion is unstable.

Finally we conclude that the electric field considered in this paper plays a significant role in controlling instability of a parallel poorly conducting fluid and hence favorable to reduce the inhomogeneity in synthesizing smart materials.

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